Estimating and Testing Two Consumption-Based Asset Pricing Models for Brazil: An Information-Theoretic Approach

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ABSTRACT: Since Brazilian data sets for consumption and asset returns are short and the standard GMM-based overidentifying restrictions test has low power in small samples, a GMM approach imposes difficulties to the evaluation of asset pricing kernels better suited to describe asset pricing phenomena in Brazil. This paper addresses the question of estimating and testing two asset pricing models, using an information-theoretic method of moments estimator, which minimizes the Kullback-Leibler Information Criterion (KLIC). The goal is to compare the traditional GMM method with the alternative information-theoretic approach, which has promising finite sample properties, focusing on over identifying restrictions test and parameter estimate.

Keywords: KLIC estimation, GMM, Asset Pricing.
1. INTRODUCTION

This paper compares two alternative approaches for estimating and testing two consumption-based asset pricing models using Brazilian data. Issler and Piqueira (2000) applied the standard GMM approach to estimate three types of asset pricing kernels for Brazil. Using quarterly data, they were unable to reject constant relative risk aversion and external habit persistence models based up on the standard over identifying restrictions test. They mention the low power of the overidentifying restrictions test as a possible explanation for their inability in rejecting these asset pricing kernels.

Several Monte Carlo simulations have pointed out that Hansen’s GMM suffer from poor finite sample properties. Hansen, Heaton & Yaron (1996) and Smith (1999) show that, infinite sample, the behavior of important statistics based up on standard GMM procedures are far from the predictions supported by first order approximation asymptotics.

The econometric literature has evolved in the direction of studying variants of the method of moments, which could provide applied researchers with econometric procedures characterized by better finite sample properties.

Empirical likelihood tests are candidates to be used as an alternative to Hansen’s GMM and are potentially capable of doing a better job in small samples. By using the Kullback-Leibler information criterion (KLIC) as a measure of distance between two distributions, it is possible to formulate an information-theoretic method of moments estimate or within the class of empirical likelihood estimators. The information-theoretic estimation strategy will be discussed in the third section.

It is important to stress that a set of Monte Carlo studies gives support to a superior small sample size-adjusted power of information-theoretic tests compared to the traditional GMM approach. Kitamura and Stutzer (1997) and Imbens et al. (1998) are seminal papers studying both the asymptotics and finite sample behavior of empirical likelihood methods. Gregory et al. (2002) conducted a Monte Carlo study in the context of asset pricing models and applied the information-theoretic KLIC estimator to test two asset pricing kernels (constant absolute and constant relative risk aversion specifications). The aforementioned papers generate evidence of good finite sample properties concerning the KLIC estimator.

The aim of this paper is to conduct estimation and inference using the information-theoretic method of moments strategy as discussed in Kitamura and Stutzer (1997) and compare the empirical results with their counterparts coming from a standard GMM framework. The focus is going to be on parameter estimates and over identifying restrictions test.

The paper is organized in four additional sections. The second describes the construction of the data set. The third discusses the information-theoretic method of moment sand its relationship with standard GMM. In the fourth section, a comparison between point estimates and overidentifying restrictions test results generated by Hansen’s GMM and the information-theoretic estimator will be presented. The last section concludes.
2. THE DATA SET

The data used are from IPEA DATA\(^1\) and from BOVESPA. The data set constructed ranges from 1974 to 1999 at quarterly frequency.

Raw data collected from IPEA DATA are: the price index IGP-DI, annual population, final household consumption and a risk-free interest rate series (Taxa Over/Selic). The IBOVESPA returns are used as the risky as-set.

The household consumption was not seasonally adjusted.

The IBOVESPA and the Taxa Over/Selic were originally monthly series and they were accumulated in order to generate data available at quarterly frequency.

The IGP-DI was used since it was available since 1944. A comparison between variables deflated using the IGP-DI and the INPC, available only since 1979 and theoretically better suited to deflate consumption series, was one. The properties of real consumption and real rate of returns using alternative deflators are very similar.

Issler and Piqueira (2000) employed the IGP-DI and Domingues (2000) reported that employing IGP-DI or the INPC index did not induce any big change in the properties of real consumption and real rate of return series. Therefore, the risk-free and risky asset returns were deflated using the IGP-DI.

The strategic task in building the data set is the construction of the real per capita consumption series, since quarterly data for household final consumption is not available before 1991.

The first step is the construction of the real consumption series. After that, in order to compute per capita figures, annual population data was used and a simple linear interpolation was able to generate quarterly population series. The errors associated with the linear interpolation should not be very high since the original annual population series is very smooth.

The discussion on how one can get quarterly real consumption series from 1974 to 1990 is in order.

The limitation faced is that quarterly data for household final consumption begins only in 1991. The same series at annual frequency is available since 1947. That information from 1974 to 1990 can be used in trying to find an approximation for quarterly missing figures.

The idea is to estimate the missing quarterly data from 1974 to 1990 using a general Kalman Filter approach. The method for estimating missing observations employing a state space framework was originally proposed by Harvey and Pierse (1984). An application of this methodology to estimate monthly GDP series based on quarterly figures for Switzerland was done by Cuche and Hess (2000).

The Econometric framework for estimating the missing quarterly data can be written generally as follows:

\[ s_{t+1} = F s_t + C' x_{t+1} + v_{t+1} \]

\[ y_t^{+} = h_t' s_t \]

The first equation describes the vector dynamics of the unobserved quarterly real consumption series \((y_t)\).

The state can be written as: \(s_t = \{y_t y_t - 1 y_t - 2 y_t - 3\}'\)

\(^1\)www.ipeadata.gov.br
*x* is any series related to the unobserved variable and *ipso facto* potentially capable of improving the estimation of the unobserved component.

*yt* is the observed series for real consumption. For *t* = 1974 to 1990, the annual data series is used. For *t* = 1991 to 1999, the available quarterly real consumption series is employed as observed figures. The annual and quarterly real consumption series were constructed by deflating the respective nominal series, using the IGP-DI.

The coefficients *h'* are defined as follows:

- For *t* = 1974 to 1990, *h'* = [1 1 1 1]
- For *t* = 1991 to 1999, *h'* = [1 0 0 0]

The definition above is capable of taking into account the aggregation restriction involving annual consumption and quarterly consumption. From 1991 to 1999, it is not necessary to impose the restriction since the quarterly consumption series is now observed.

The Econometric framework described above was used in two ways. First, an autoregressive structure was imposed to *yt* and no additional series was used to improve upon the estimation of *yt*. Second, the estimation of the missing quarterly figures was done using residential consumption of energy and industrial production of consumption goods as related series in *xt*. The consumption of energy is a monthly series and is available since 1976 and the industrial production of consumption goods is a quarterly series available since 1975. So, in the second way to estimate the missing figures, the first observation used started at 1976.

The two ways of estimating the missing quarterly values are equivalent qualitatively. Therefore, the outcome of the first approach is adopted as the real consumption series to be employed in the construction of the per capita real consumption series, which is going to be finally used for estimation purpose.

The first three quarters of per capita real consumption data was discarded. The dataset employed in the following estimations ranges from 1974 (Q4) to 1999 (Q4).

Table 1 below summarizes basic statistics associated with the series that will be employed in the estimation of asset pricing kernels. *Rf* is the real Taxa Over/Selic gross return, the risk-free rate of return. *Ribo* is the real IBOVESPA gross return, which measures the risky rate of return. Finally, *Cg* denotes the per capita real consumption growth series.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cg</td>
<td>1.000681</td>
<td>0.067772</td>
<td>3.307907</td>
<td>27.67400</td>
</tr>
<tr>
<td>Rf</td>
<td>1.013707</td>
<td>0.083953</td>
<td>-1.595800</td>
<td>10.47141</td>
</tr>
<tr>
<td>Ribo</td>
<td>1.047722</td>
<td>0.257595</td>
<td>0.587581</td>
<td>3.725827</td>
</tr>
</tbody>
</table>

Figures 1, 2 and 3 display the time series plot of per capita real consumption, *Ribo* and *Rf* respectively.
3. ESTIMATION AND TEST PROCEDURES

3.1 The information-theoretic Method of Moments

This section presents the information-theoretic method of moments which minimizes the KLIC distance and discusses its relationship with Hansen’s.

![Figure 1: Real Consumption](image1)

![Figure 2: Ibovespa Return](image2)
GMM:
Let $\theta$ be the parameter vector of a general asset pricing model. Moment conditions can be generated by the choice of appropriate instruments in conjunction with economic theory and can be written as

$$E_{v}[f_{i}(y, \theta_{0})] = \int [f_{i}(y, \theta_{0})] dv(y) = 0$$

for $i = 1, 2, ..., m$.

Here $y$ is a vector of data points, the total number of observations is $T$. The number of moment conditions equals $m$.

$\theta_{0}$ is the parameter vector to be estimated and $E_{v}$ is the expectation with respect to the probability measure $v$. The empirical analog will be:

$$f_{i,T}(\theta) = \sum_{t=1}^{T} \frac{1}{T} f_{i}(y_{t}, \theta)$$

for $i = 1, 2, ..., m$.

Let $f_{T}(\theta)$ be the vector of moments with the component $I$ equals to $f_{i}$. The GMM estimator is:

$$\hat{\theta}_{GMM} = \arg\min_{\theta} f_{T}(\theta)' W f_{T}(\theta)$$

$W$ is the weighting matrix, computed from the inverse of the asymptotic variance of moment conditions. For more details on GMM see the textbook discussion in Hamilton (1994).

Kitamura and Stutzer (1997) proposed an alternative to Hansen’s GMM based upon the following minimization problem.

$$\min_{P_{EM}} D(P, v) = \int \log \left( \frac{dP}{dv} \right) dP$$

Subject to: $E_{v}[f(y, \theta)] = 0$ para $i = 1, 2, ...., m$. 

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**Figure 3: Risk Free Return**

![Real Risk Free Gross Return](image)

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The expression denoted by \((P,v)\) measures the Kullback-Liebler distance between the two measures and is zero only if \(P = v\). If there is no parameter vector satisfying the moment conditions, then the model is rejected and \(P \neq v\).

The KLIC estimation procedure searches over the parameter space to make measure \(P\), which belongs to the admissible measures \(M\), as close as possible to \(v\).

The sample version of this optimization problem is:

\[
\sum_{t=1}^{T} \log(Tp_t)p_t
\](7)

subject a:

\[
\sum_{t=1}^{T} f_t(y_t, \theta)p_t = 0 \quad \text{for } i = 1, 2, \ldots, m
\]

The expression for \(p_t\) is:

\[
p_t = \frac{\exp(t'f(y_t, \theta))}{\sum_{t=1}^{T} \exp(t'f(y_t, \theta))}
\](8)

Where \(t\) is a \(m\)-vector which can be interpreted as the Lagrange multipliers in the minimization problem above.

GMM focus attention on the inability of parameters different from the true parameter value to satisfy moment conditions defined in terms of the original measure \(v\).

The estimation discussed here propose a change of measures which enables any parameters different from the true one to satisfy the moment conditions now defined in terms of a new measure \(P\).

Of course, only the true parameter value is able to satisfy the moment conditions associated with the original measure \(v\). Therefore, the transformed measure \(P\) associated with the true parameter vector is essentially equal to the original measure \(v\) in a statistical sense. With this is mind, it makes sense to search for a parameter that minimizes the distance between the transformed measure and the original one.

To implement the method described above, first it is necessary to smooth the moment conditions. The unsmoothed estimator is still consisted but is not asymptotically efficient. The smoothed moment conditions can be written as follows:

\[
f_{si}(\theta) = \sum_{t=1}^{T} \frac{1}{2K+1} f_t(y_t-k, \theta)
\](9)

for \(i = 1, 2, \ldots, m\)

The information-theoretic estimator can be computed as follows:

\[
\hat{\theta}, \hat{t} = \arg \max_{\theta} \min_{t} \{ Q(\theta, t) = \frac{1}{T} \sum_{t=1}^{T} \exp(t'f_{s}(\theta, t))\}
\](10)

The information-theoretic estimator is consistent and, to first order, is asymptotically equivalent to the standard GMM estimator. Therefore the asymptotic normality results associated with GMM are still valid for the KLIC estimator. Proofs of these results and a deep technical discussion can be found in Kitamura and Stutzer (1997) and in Imbens et al. (1998).

In sum, the asymptotic distribution of \(\hat{\theta} \hat{t}\):
where $V$ is given by:

$$V = \left\{ E\left( \frac{\partial f_s(y, \theta)}{\partial \theta} \right)' \left[ E(f_s(y, \theta)fs(y, \theta)') \right]^{-1} E\left( \frac{\partial f_s(y, \theta)}{\partial \theta} \right) \right\}^{-1}$$

A consistent estimate of the matrix $V$ can be constructed by using empirical analogs of the expectations above in which each data point has to be weighted by the estimated $\hat{\theta}_n$.

A test of overidentifying restrictions can be built using the following test statistics:

$$- \frac{2T}{2K + 1} \log(\hat{Q}(\theta, t))$$

Under null hypothesis (3), the test statistics above has a chi-squared distribution with degrees freedom given by: number of moment conditions minus number of parameters to be estimated, i.e., $m$ minus the number of elements in the parameter $\theta$.

An excessively large value for the test statistics leads to the rejection of the null, as usual in chi-squared type of tests.

Of course, it is possible to construct Wald, Lagrange Multiplier and Likelihood Ratio-like test statistics based upon the information-theoretic method of moments proposed by Kitamura and Stutzer (1997).

Suppose we have $nr$, possibly nonlinear restrictions, involving the parameters of an asset pricing model. These restriction can or cannot hold in the model with the true parameter vector. Therefore, it is important to test the following null hypothesis $H_0: R(\theta) = a$. Note that $nr$ is less than the number of elements in the parameter vector $\theta$.

The Wald statistics is given by:

$$Wald = T [(R(\hat{\theta}) - a)' [A^T V^{-1} A]^{-1} (R(\hat{\theta}) - a)]$$

The Wald statistics is asymptotically chi-squared distributed with degrees of given by $nr$.

$A_T$ is a consistent estimate of Jacobian matrix of $R$ and $V_T$ is a associated with estimate of $V$.

The Wald test will be used later to evaluate some parametric restrictions associated with asset pricing kernels.

### 3.2 Asset Pricing Kernels

Consumption-based asset pricing models are based upon portfolio and consumption choices of a representative individual in a dynamic and stochastic setting.

The Euler Equation coming from the representative consumer optimization problem, can be written as follows:

$$1 = E_t[(1 + r_{it+1})M_{t+1}]$$

According to the type of preference considered $M_{t+1}$, the stochastic discount factor, will assume different functional specifications.

In this paper, only two asset pricing kernels are the subject of study.

- The asset pricing kernel associated with constant relative risk aversion preferences (CRRA), characterized by the power utility:
The Euler equation in this case is given by:

$$u(C_t) = \frac{C_t^{1-\gamma} - 1}{1 - \gamma}$$  \hspace{1cm} (15)

The Euler equation in this case is given by:

$$E_t[\beta(1 + r_{it+1})\left(\frac{C_{t+1}}{C_t}\right)^{k(1-\gamma)}] = 1$$ \hspace{1cm} (16)

- The asset pricing kernel associated with external habit persistence preferences, given by the following utility specifications:

$$u(C_t) = \frac{(\frac{C_t}{X_t})^{1-\gamma} - 1}{1 - \gamma}$$ \hspace{1cm} (17)

The variable $X_t$ is defined as follows $X_t = \frac{C_t}{C_{t-1}}^k$, where $\frac{C_t}{C_{t-1}}^k$ is the aggregate past consumption. The parameter $k$ governs the degree of time-nonseparability.

The Euler equation for this class of preference is going to be:

$$E_t[\beta(1 + r_{it+1})\left(\frac{C_{t+1}}{C_t}\right)^{k(1-\gamma)}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}] = 1$$ \hspace{1cm} (18)

The important parameters for the two kernels considered are: the coefficient of relative risk aversion $\gamma$, intertemporal discount factor $\beta$ and the time-nonseparability parameter $k$.

These asset pricing kernel are discussed more extensively in chapter 8 of Campbell et al. (1997).

3.3 The Applied Econometric Exercise

It is worth describing the empirical procedure adopted in this paper.

First, Hansen’s GMM and the KLIC-based method of moments are used to estimate the asset pricing models mentioned in the previous sub-section.

The Bartlett kernel with smoothing parameter $k = 6$ was employed in constructing the optimal weighting matrix to perform GMM estimation. In order to compare GMM and KLIC-based estimators, the same number $k = 6$ was used to smoothing the moment conditions to implement the information-theoretic approach, according to (9).

Four sets of instruments are used for each asset pricing model considered and they are detailed in tables 2 and 3 below.

| Table 2. Instruments - CRRA |
|----------------------------|--|
| First Set                  |
| $\text{cons}$ $\text{Ribo}_t$ |
| Second Set                 |
| $\text{cons}$ $\text{Ribo}_t$ |
| Third Set                  |
| $\text{cons}$ $\text{Ribo}_t$ |
| Fourth Set                 |
| $\text{cons}$ $\text{Ribo}_t$ |

<table>
<thead>
<tr>
<th>Table 3. Instruments – External Habit</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Set</td>
</tr>
<tr>
<td>$\text{cons}$ $\text{Ribo}<em>t$ $\frac{C</em>{t-1}}{C_t}$ $\frac{C_{t-2}}{C_t}$ $\frac{C_{t-3}}{C_t}$</td>
</tr>
</tbody>
</table>
Estimating and Testing Two Consumption-Based Asset Pricing Models for Brazil

Notation: cons denote a constant, Ribo is the IBOVESPA gross return. Additionally, consumption growth with different lags are included as instruments.

After the estimation stage is done, the second step is to perform overidentifying restrictions tests, which will address the capability of the asset pricing kernel to cope with the empirical regularities in the data.

Third, some parametric restrictions will be tested using the Wald Test in order to gather evidence agent to validate some particular characteristic associated with the representative agent utility function.

Are pointed out before, the basic issue is to compare results coming form different estimation strategies. The questions of interest are:

- Do both methods give support to or reject specific qualitative features associated with particular asset pricing specifications?
- Do the methods reject or give support to specific features with the same intensity? For instance, do they reject the same feature equally strongly or one method rejects a particular feature more strongly than the other?
- Is the any unanimous characteristic related to the representative agent preferences, which is strongly validated by both methods?

The next section summarizes the empirical findings and answers the question above:

4. EMPIRICAL RESULTS

4.1 Parameter Estimates and Overidentifying Restrictions Tests
- Standard GMM

The following tables show parameter estimates and t statistic with their p-values.

<table>
<thead>
<tr>
<th>Instruments</th>
<th>First</th>
<th>Second</th>
<th>Third</th>
<th>Forth</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>0.9875</td>
<td>0.9889</td>
<td>0.9901</td>
<td>0.9908</td>
</tr>
<tr>
<td>( t ) statistics (( \beta ))</td>
<td>113.815</td>
<td>108.562</td>
<td>127.404</td>
<td>123.102</td>
</tr>
<tr>
<td>p-value</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>( \Gamma )</td>
<td>0.6353</td>
<td>0.5577</td>
<td>0.5383</td>
<td>0.5801</td>
</tr>
<tr>
<td>( t ) statistics (( \gamma ))</td>
<td>2.6149</td>
<td>2.5673</td>
<td>2.6703</td>
<td>2.7082</td>
</tr>
<tr>
<td>p-value</td>
<td>0.0096</td>
<td>0.0011</td>
<td>0.0082</td>
<td>0.0074</td>
</tr>
</tbody>
</table>

Table 5. GMM Estimates – External Habit

<table>
<thead>
<tr>
<th>Instruments</th>
<th>First</th>
<th>Second</th>
<th>Third</th>
<th>Forth</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>0.9896</td>
<td>0.9859</td>
<td>0.9933</td>
<td>0.9890</td>
</tr>
<tr>
<td>( t ) statistics (( \beta ))</td>
<td>115.98</td>
<td>95.48</td>
<td>131.29</td>
<td>125.56</td>
</tr>
</tbody>
</table>
• Information-Theoretic Method of Moments

The following tables show parameter estimates and t statistics with their p-values.

**Table 6. KLIC - CRRA Estimates**

<table>
<thead>
<tr>
<th>Instruments</th>
<th>First</th>
<th>Second</th>
<th>Third</th>
<th>Forth</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.9810</td>
<td>0.9919</td>
<td>0.9886</td>
<td>0.9916</td>
</tr>
<tr>
<td>$t$ statistics ($\beta$)</td>
<td>41.95</td>
<td>32.58</td>
<td>55.79</td>
<td>36.78</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>0.7223</td>
<td>0.9566</td>
<td>0.6590</td>
<td>0.8680</td>
</tr>
<tr>
<td>$t$ statistics ($\gamma$)</td>
<td>1.2285</td>
<td>1.2331</td>
<td>1.6648</td>
<td>1.3670</td>
</tr>
<tr>
<td>p-value</td>
<td>0.1096</td>
<td>0.1080</td>
<td>0.0480</td>
<td>0.0858</td>
</tr>
</tbody>
</table>

**Table 7. Estimates KLIC – External Habit**

<table>
<thead>
<tr>
<th>Instruments</th>
<th>Firth</th>
<th>Second</th>
<th>Third</th>
<th>Forth</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.9934</td>
<td>1.0001</td>
<td>1.0093</td>
<td>0.9713</td>
</tr>
<tr>
<td>estatísticas t ($\beta$)</td>
<td>37.05</td>
<td>30.52</td>
<td>56.07</td>
<td>45.28</td>
</tr>
<tr>
<td>valor p</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>0.8455</td>
<td>1.1716</td>
<td>0.8781</td>
<td>0.9239</td>
</tr>
<tr>
<td>estatísticas t ($\gamma$)</td>
<td>1.4222</td>
<td>0.4607</td>
<td>1.3448</td>
<td>1.3762</td>
</tr>
<tr>
<td>valor p</td>
<td>0.0775</td>
<td>0.3225</td>
<td>0.0893</td>
<td>0.0844</td>
</tr>
<tr>
<td>$K$</td>
<td>-0.2209</td>
<td>0.6572</td>
<td>-1.9980</td>
<td>-1.2006</td>
</tr>
<tr>
<td>estatísticas t ($k$)</td>
<td>-0.0370</td>
<td>0.0171</td>
<td>-0.1628</td>
<td>-0.0953</td>
</tr>
<tr>
<td>valor p</td>
<td>0.5148</td>
<td>0.4932</td>
<td>0.5647</td>
<td>0.5380</td>
</tr>
</tbody>
</table>

Overidentifying restrictions test results are summarized bellow for each econometric method studied. The Test Statistics are chi-squared distributed and their values are shown in the following tables with respective p-values.
Table 8. CRRA

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>First</td>
<td>3.2230</td>
<td>6.5450</td>
<td>0.1620</td>
<td>3.2765</td>
<td>4.6347</td>
<td>0.3047</td>
</tr>
<tr>
<td>Second</td>
<td>3.5287</td>
<td>8.2231</td>
<td>0.2222</td>
<td>3.2069</td>
<td>5.2574</td>
<td>0.5112</td>
</tr>
<tr>
<td>Third</td>
<td>3.2069</td>
<td>5.2574</td>
<td>0.7824</td>
<td>3.2069</td>
<td>5.2574</td>
<td>0.7824</td>
</tr>
<tr>
<td>Forth</td>
<td></td>
<td></td>
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</table>

Table 9. External Habit

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<thead>
<tr>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>2.9085</td>
<td>6.5450</td>
<td>0.5059</td>
<td>3.7409</td>
<td>9.9586</td>
<td>0.1264</td>
</tr>
<tr>
<td>Second</td>
<td>4.3328</td>
<td>11.4237</td>
<td>0.0761</td>
<td>2.0880</td>
<td>6.6132</td>
<td>0.3581</td>
</tr>
<tr>
<td>Third</td>
<td>2.0880</td>
<td>6.6132</td>
<td>0.8368</td>
<td>2.0880</td>
<td>6.6132</td>
<td>0.8368</td>
</tr>
<tr>
<td>Forth</td>
<td></td>
<td></td>
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</tbody>
</table>

4.2 Additional Tests

For the external habit specification, the Wald Test was done to test the following null hypothesis: $H_0: \gamma = 0$ and $k = 0$.

The results are summarized in Table 10. GMM strongly rejects the hypothesis, with very high Wald Statistics, but the KLIC-based method gives mixed evidence against null.

Table 10. External Habit

<table>
<thead>
<tr>
<th>Instruments</th>
<th>Firth</th>
<th>Second</th>
<th>Third</th>
<th>Forth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wald GMM</td>
<td>9.0402</td>
<td>53.8200</td>
<td>2190.3550</td>
<td>146.4462</td>
</tr>
<tr>
<td>p-value</td>
<td>0.0108</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Wald KLIC</td>
<td>2.1371</td>
<td>4.6525</td>
<td>29.3027</td>
<td>29.2591</td>
</tr>
<tr>
<td>p-value</td>
<td>0.3435</td>
<td>0.0977</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

In addition, the Wald Test was used, in cases where the estimated discount factor exceeded one, to test the following null hypothesis: $H_0: \beta = 0.98$.

The null was tested under KLIC-based estimation method, employing the second and third set of instruments, the statistic and their p-values in parenthesis are, respectively 0.03727(0.5416) and 2.7214(0.0990). At 5%, the null cannot be rejected. Therefore, it is possible that the discount factor is less than one these specifications.

4.3 Comparative Remarks

Concerning parameter estimates, both methods agree that external habit formation is not present in the representative agent utility function since $k$ is not significant under both estimation procedures and the null $H_0: k = 0$ cannot be rejected very strongly.

The discount factor is very high but bellow one, even though it exceeds on for two sets of instruments under the estimation contucted via KLIC-based method of moments.

The big disagreement concerns the value of the relative risk parameter $\gamma$. This parameter was found to be significant under GMM and this result is very strong. Employing
the information-theoretic estimation strategy, $\gamma$ was not significant at 5% though it was significant at 10%. There is evidence, though not a strong one, favoring the following preference $u(C_t) = \log(C_t)$ for the representative agent. The Wald Statistic testing evidence supporting the log utility specification. In sum, $u(C_t) = \log(C_t)$ seems to be empirically supported by KLIC-based estimation, though not strongly. This result stand in contrast with rejects this utility specification very strongly.

Concerning overidentifying restrictions test results, both models are not rejected. Therefore, the results in Issler and Piqueira (2000) holds even if the estimation method used is the KLIC-based method of moments.

One important remark is related to the magnitude of p-values as a measure of how strongly the overidentifying restrictions could not be rejected. Usually, p-values coming from the standard GMM. It is an indication that the KLIC-based method is more prone to reject the null than the standard GMM. This observation is in line with Monte Carlo studies pointing out to a better size-adjusted power associated with the KLIC-based method of moments. In sum, Issler and Piqueira (2000) result seems to be robust and the power utility as well as the external habit specification could not be rejected also by test prone to rejected mor Hansen’s GMM.

5. FINAL COMMENTS

The goal of this paper was to estimate and test two asset pricing specifications using the information-theoretic method of moments strategy as discussed in Kitamura and Stutzer (1997). A comparative analysis between empirical findings obtained via information-theoretic method of moments and their counterparts coming from standard GMM was done.

Summing up, the existence of habit formation was rejected by both econometric procedures though both asset pricing kernels considered could not be rejected by overidentifying restrictions test. The KLIC-based estimation strategy gives some support to the following specification $u(C_t) = \log(C_t)$. By contrast, GMM gives empirical support to the traditional power utility specification with $\gamma$ between 0.5 and 1.2.

In order to complement this work, a clear understanding of the finite sample properties of the estimators studies is needed. We hope to incorporate this analysis, considering the particular data generating process associated with Brazilian consumption and asset returns, in future research.

REFERENCES


IMBENS, Guido et. al. (1998) "Information Theoretic Approaches to Inference in Moment Condition Models”. Econometrica, v.66, 2, 333-357.
