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The Empirical Relationship between Stock Returns, Return Volatility and Trading Volume in the Brazilian Stock Market

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ABSTRACT: We investigate the empirical relationship between stock returns, return volatility and trading volume in the Brazilian stock market (Bovespa). Our sample contains stock return and trading volume data from a theoretical portfolio including stocks participating in the Bovespa Index (Ibovespa) extending from 01/03/2000 through 12/29/2005. The empirical methods used include cross-correlation analysis, unit-root tests, bivariate simultaneous equations regression analysis, GARCH and VAR models, and Granger causality tests. We find support for a contemporaneous as well as a dynamic relationship between stock returns and trading volume, implying that forecasts of one of these variables can be only slightly improved by knowledge of the other. Besides, our results indicate that contemporaneous and dynamic relationships between return volatility and trading volume also exist. Additionally, by applying Granger's causality test, we find that return volatility contains information about upcoming trading volume and vice versa.

Key words: return volatility; trading volume; GARCH; VAR; Granger causality.

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1. INTRODUCTION

Empirical studies on stock markets usually focus on stock prices and their behavior over time. However, due to some undesirable stochastic properties of stock prices, especially non-stationarity, most researchers concentrate on stock returns rather than prices. Based on the existing information about a firm, its stock returns reflects the investors' expectations on the future performance of that firm. The arrival of new information makes investors to adapt their expectations and this is the main cause for price and return changes. However, since investors are heterogeneous when interpreting new information, stock returns may stay unchanged even though new information is brought to the market. This will be the case if some investors interpret some bits of information as good news while others find it to be bad news. Therefore, price changes indicate the average reaction of investors to news. On the other hand, stock returns may only change if there is positive trading volume. As it happens with returns, trading volume and its changes mainly reflect the available set of relevant information perceived by the market. Differently from stock prices and returns, however, a relevant change in investors' expectations always leads to an increase in trading volume which therefore reflects the sum of investors' reactions to news. Studying the joint dynamics of stock returns and trading volume therefore improves the understanding of the microstructure of stock markets.

Our study is inspired on a previous work which investigates the relation between stock returns, return volatility and trading volume in the Austrian stock market (MESTEL; GURGUL; MAJDOSZ, 2003). Our investigation cover not only contemporaneous but also dynamic relationships. Our results indicate that there is an association between stock returns and trading volume in the Brazilian stock market. This implies, *inter alia*, that knowledge of trading volume may improve short-run return forecasts. Moreover, we find support for the hypothesis of a positive relationship between return volatility and trading volume. Actually, this positive relationship shows evidence of a mutual Granger causality between return volatility and trading volume.

The paper is organized as follows: In section 2 we present a brief discussion on the extant literature and in Section 3 we describe the methods used in our empirical investigation. In section 4 we show our findings with respect to the contemporaneous and the dynamic relationship between stock returns, return volatility and trading volume. Section 5 concludes the paper.

2. LITERATURE REVIEW

Several studies have examined the empirical relationship between trading volume and stock returns. Some of these have investigated the connection between trading volume and price changes by itself, usually using price indexes (KARPOFF, 1987; HIEMSTRA and JONES, 1994; BRAILSFORD, 1996; LEE and RUI, 2002). The results of these studies diverge from each other, although a positive relationship is mainly reported. Also, the association between stock return volatility and trading volume has been analyzed by several authors since the 1980's (KARPOFF, 1987; BROCK; LEBARON, 1996; LEE; RUI, 2002; MESTEL; GURGUL; MAJDOSZ, 2003).

Recently, stochastic time series models of conditional heteroscedasticity have been used to explore this relationship (LAMOUREUX; LASTRAPES, 1990; ANDERSEN, 1996; BRAILSFORD, 1996; GALLO; PACINI, 2000; OMRAN; MCKENZIE, 2000). Most of these studies mostly conclude that there is evidence of a strong relationship, which is both contemporaneous as well as dynamic, between return volatility and trading volume. However, there is reported evidence using intraday data from the Dow Jones Industrial Average stocks of only significant lead/lag relations but not of contemporaneous correlation between return volatility and trading volume (DARRAT; RAHMAN; ZHONG, 2003).

A recent study scrutinizes the empirical relationship between stock returns, return volatility and trading volume on the Austrian stock market (MESTEL; GURGUL; MAJDOSZ, 2003). The authors find only weak support for a contemporaneous and dynamic relationship between stock returns and trading volume, implying that forecasts of one of these variables cannot be improved by knowledge of the other. However, they find evidence of a strong contemporaneous relationship between return volatility and trading volume and that return volatility contains information about upcoming trading volume.

3. METHODOLOGY

3.1. Sample and Data

Our data set comprises daily market price and trading volume series for a theoretical portfolio consisting of assets belonging to all (57) firms participating in Ibovespa, the Brazilian stock-exchange (Bovespa) index. Overall, these assets account for over 80% of Bovespa's market capitalization. The investigation covers the period extending from 01/03/2000 to 12/29/2005. Ibovespa is a market-capitalization weighted stock index that indicates the total return performance of all securities traded in the prime market segment of Bovespa and works as a benchmark for institutional investors.

All trading volume and stock index data are primarily provided by Bovespa and were collected from Economatica®'s database. Continuously compounded stock returns are calculated from daily stock prices at close, adjusted for dividend payouts and stock splits. Since Brazilian inflation is not negligible, around 11.5% per year during the 2001-2005 period, according to the Brazilian general price index, IGP-DI, we deflate the trading volume and the Ibovespa series using the IGP-DI itself. The data on the IGP-DI were collected from IPEA – Instituto de Pesquisa Econômica Aplicada's website (IPEA, 2005). Since trading volumes in Brazilian currency (BRL$) are quite large numbers, we transformed the series into an index, with the first observation being set to 100.

3.2. Cross-Correlation Analysis

As a first step to investigate the relationship between stock return and trading volume, we calculate the cross-correlation coefficients $\rho(R_t,V_t)$ for all firms:

$$\rho(R_t,V_t) = \frac{\mathrm{Cov}(R_t,V_t)}{\sigma(R_t)\cdot\sigma(V_t)} \qquad (1)$$

where R_t and V_t stand for stock return and trading volume, respectively, on day t, Cov(·,·) denotes covariance and σ (·) is standard deviation.

3.3. Testing for Unit Roots

To test for the contemporaneous as well as causal relation between trading volume, stock returns and return volatility, we use an unrestricted Vector Autoregressive (VAR) model that can be sensitive to non-stationarity. Therefore we check the hypothesis of whether the time series of stock returns and trading volume are stationary by using the augmented Dickey-Fuller (ADF) test. This test is based on the regression:

$$\Delta y_t = \mu + \gamma\, y_{t-1} + \sum_{i=1}^{p} \delta_i \Delta y_{t-i} + \varepsilon_t \qquad (2)$$

where y is the variable being tested for unit roots (stock return or trading volume), μ, γ and δ are model parameters and ε_t is an I.I.D Gaussian (0, σ^2) white noise error term.

The unit root test is carried out by testing the null hypothesis $\gamma = 0$ against the one sided alternative $\gamma < 0$. The t-Student-statistic of the estimated parameter γ does not have a conventional t-distribution under the null hypothesis of a unit root. Instead, we use the critical values recommended by MacKinnon (1991). If the ADF t-statistic for γ lies to the left of these values, the null hypothesis can be rejected.

3.4. Stock Returns and Trading Volume

The empirical procedure in this section further tests the contemporaneous relationship between stock returns and trading volume. We apply the multivariate model proposed in a previous work, which is defined by the two equations below (LEE; RUI, 2002):

$$R_t = \alpha_0 + \alpha_1 V_t + \alpha_2 V_{t-1} + \alpha_3 R_{t-1} + u_t \qquad (3)$$

$$V_t = \beta_0 + \beta_1 R_t + \beta_2 V_{t-1} + \beta_3 V_{t-2} + v_t \qquad (4)$$

where α_i and β_i , i = 1, 2, 3, are model coefficients and u_t and v_t, denote I.I.D Gaussian (0, σ^2) white noise error terms. To estimate the model coefficients we apply the full-information maximum likelihood method.

Although we may find stock return levels and trading volume to be mostly uncorrelated that does not mean that there is no relationship between these variables at all. It is often reported that price fluctuations tend to increase if there is a high trading volume, especially in times of bullish markets. That is, there might be a relation between higher order moments of stock returns and trading volume. We scrutinize this by extending a model which relates trading volume to squared stock returns by means of the following regression (BRAILSFORD, 1996):

$$V_t = \alpha_0 + \phi_1 V_{t-1} + \phi_2 V_{t-2} + \alpha_1 R_t^2 + \alpha_2 D_t R_t^2 + e_t \qquad (5)$$

where D_t denotes a dummy variable that equals 1 if the corresponding return R_t is negative and 0 otherwise. To avoid the problem of serially correlated residuals documented in Brailsford (1996) we include lagged values of V_t up to lag 2. The estimate of parameter α_1 measures the relationship between return volatility and trading volume irrespective of the direction of the price change. The estimate of α_2, however, measures the degree of asymmetry in that relationship.

3.5. Conditional Volatility and Trading Volume

Eventually, the finding of a contemporaneous relationship between trading volume and squared stock returns raises the question of whether trading activity can be identified as a potential source for the observed serial dependence (persistence or hysteresis) in return volatility. This is motivated by the theoretical works on the Mixture of Distribution Hypothesis (MDH) (CLARK, 1973; EPPS; EPPS, 1976; TAUCHEN; PITTS, 1983; LAMOUREUX; LASTRAPES, 1990; ANDERSEN, 1996). This hypothesis affirms that stock returns are generated by a mixture of distributions in which the number of information arrivals into the market represents the stochastic mixing variable. Return data can be regarded as a stochastic process, conditional on the information flow, with a changing second order moment reflecting the intensity of information arrivals. Under the assumptions of the MDH model, innovations to the information process lead to momentum in stock return volatility.

Since the information flow into the market is widely unobservable, we use trading volume as a proxy. Systematic variations in trading volume are assumed to be caused only by the arrival of new information. Trading volume typically exhibit the assumed time dependence. We specify the stochastic process of stock returns as a simple GARCH (1,1) process with an autoregressive term in the mean equation and trading volume as an additional predetermined regressor in the conditional variance equation:

$$R_t = \mu + \phi R_{t-1} + \varepsilon_t \tag{6}$$

$$\varepsilon_t \mid I_{t-1} \sim \text{Gaussian}(0, \sigma_t^2) \tag{7}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_t^2 + \gamma_1 V_t + \zeta_t \tag{8}$$

where I_{t-1} denotes the set of information available at t–1 and σ_t^2 stands for the variance of ε_t. The parameters of equations (7) and (8) are estimated by means of maximum likelihood. Note that in equation (8) the sum of parameters α_1 and β_1 is a measure of the persistence in the variance of the unexpected return ε_t taking values between 0 and 1. The more this sum tends to unity the greater the persistence of shocks to volatility, which is also known as volatility clustering or hysteresis.

3.6. Causal Relationship

Up to now we have mainly concentrated on the contemporaneous relationship between stock returns, return volatility and trading volume. In this section we extend our analysis by examining the dynamic (causal) relationship. Testing for causality is important since it can help to better understand the microstructure of stock markets and can also have implications for other markets (e.g. options markets).

We investigate causality between trading volume and stock returns and between trading volume and return volatility in both directions by means of Granger's causality test (GRANGER, 1969). A variable *y* is said to not Granger-cause a variable *x* if the distribution of *x*, conditional on past values of *x* alone, equals the distribution of *x*, conditional on the past of both *x* and *y*. On the other hand, if this equality does not hold, y is said to Granger-cause *x*. However, this does not mean that *y* causes *x* in the more common sense of the term but only indicates that *y* precedes *x*.

To test for Granger causality we use a bivariate VAR model of order p of the form:

$$R_t = \mu_R + \sum_{i=1}^{p} \alpha_i R_{t-i} + \sum_{i=1}^{p} \beta_i V_{t-i} + u_t \tag{9}$$

$$V_t = \mu_V + \sum_{i=1}^{p} \alpha_i V_{t-i} + \sum_{i=1}^{p} \beta_i R_{t-i} + v_t \tag{10}$$

The null hypotheses that *R* does not Granger-cause *V* and that *V* does not Granger – cause *R* imply that β_i (i = 1, …, p) are all equal to 0. To test the null we calculate the F-statistic:

$$F = \frac{SSE_r - SSE_u}{SSE_u} \times \frac{N - 2p - 1}{p} \tag{11}$$

where SSE_r stands for the sum of squared residuals of the restricted regression (i.e. $\beta_1 = \ldots = \beta_p = 0$), SSE_u is the sum of squared residuals of the unrestricted equation, and N is the number of observations. The statistic in equation (11) is asymptotically F distributed under the null with *p* degrees of freedom in the numerator and (N – 2p –1) in the denominator. The parameters α_i and β_i in equation (9) and (10) are estimated by OLS. To decide upon the appropriate order *p* of the VAR we use the adjusted R^2 and the Akaike and the Schwartz information criteria (AIC/SIC). These are measures of goodness of fit that adjust for the loss of degrees of freedom resulting from adding additional lags to the model. The bivariate regressions in (9) and (10) are re-estimated with squared values of stock returns instead of return levels.

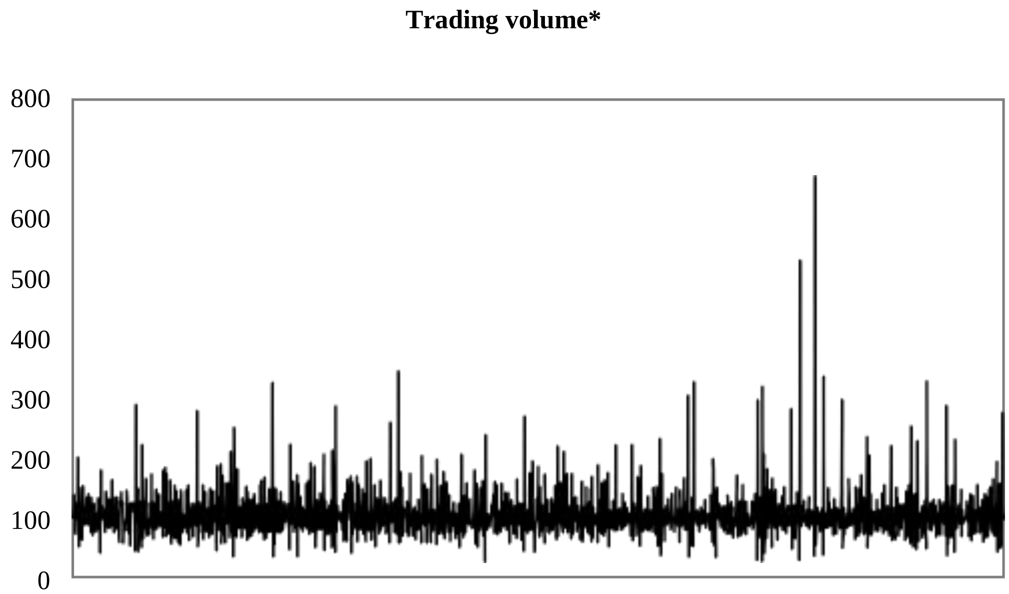
4. RESULTS

4.1. Descriptive Statistics

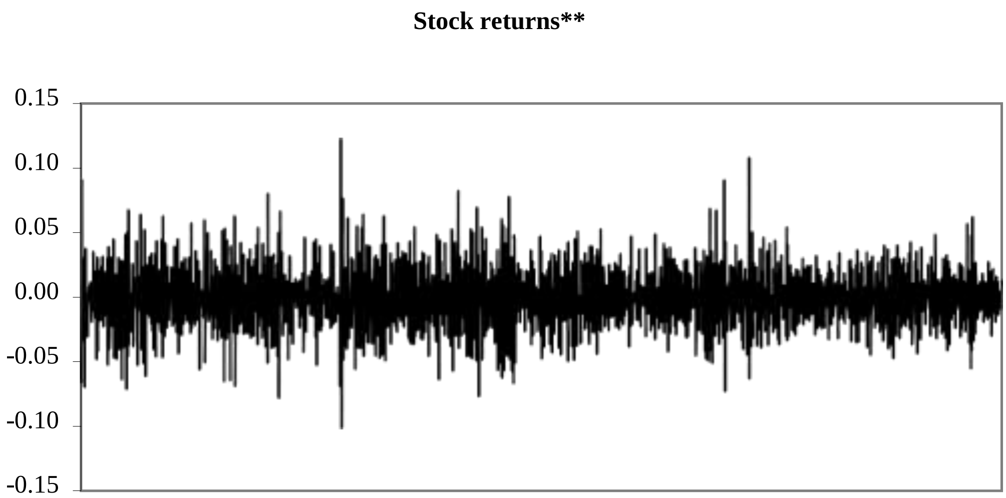
We started our investigation with some basic descriptive analysis of the time series of stock returns and trading volume, which are shown on Figure 1 and Table 1.

The mean daily stock return is equal to 0.000398%, with a standard deviation of 2.6%. The ‘fat-tailed and highly-peaked’ stylized fact that is often reported for return series is mostly present in our data. The excess kurtosis is 0.501 and the return skewness is 0.155. Applying the Jarque-Bera test for normality we find strong support for the hypothesis that the time series of stock returns do not correspond to a normal distribution.

Figure 1: Ibovespa’ trading volume and stock returns



* From 03/01/2000 to 12/29/2005, index 03/01/2000 = 100; deflated by IGP-DI; Source: Economatica®



** From 03/01/2000 to 12/29/2005; Ibovespa deflated by IGP-DI; Source: Economatica®

Unlike stock returns, return volatility as well as trading volume present strong persistence in their times series, which is investigated in this section by means of a GARCH model. Hence, in accordance with the stylized facts of volume series documented in the extant literature, our volume data show remarkably non-Gaussian characteristics, i.e. positive excess kurtosis and skeweness to the right (Andersen, 1996). In addition, we find that log-values of trading volume can be assumed to follow a normal distribution.

To proxy return volatility we use squared values of daily stock returns. These time series display the usual time dependency of stock returns in the second order moment (volatility persistence or hysteresis) implying that returns cannot be assumed to be I.I.D.

Table 1: Statistics summary

Sample: 1 1492				
	V*	R	R^2	Log(V)
Mean	951.4742	3.98E-06	0.000683	6.767235
Median	873.8831	-0.000313	0.000329	6.772947
Maximum	3991.374	0.122416	0.014986	8.291891
Minimum	171.7479	-0.101463	0.000000	5.146028
Std. Dev.	425.3271	0.026148	0.001081	0.427677
Skewness	1.590360	0.155579	4.671124	-0.072310
Kurtosis	8.436255	3.501411	40.68527	3.223403
Jarque-Bera	2464.489	21.63393	93650.80	4.399939
Probability	0.000000	0.000020	0.000000	0.110807

* BRL$ 10^6

4.2. Cross-Correlation Analysis

Table 2 presents cross-correlation coefficients between stock returns, return volatility, and trading volume.

Table 2: Cross-correlation coefficients between stock returns, return volatility and trading volume

		V(-2)	V(-1)	V	V(+1)	V(+2)	R	R^2
V(-2)	Correlação Pearson	1	-.253(**)	-.103(**)	-.084(**)	.024	-.023	-.020
	Sig. (1-tailed)	.	.000	.000	.001	.177	.186	.223
	N	1491	1490	1489	1488	1487	1489	1489
V(-1)	Correlação Pearson	-.253(**)	1	-.253(**)	-.103(**)	-.084(**)	-.081(**)	.088(**)
	Sig. (1-tailed)	.000	.	.000	.000	.001	.001	.000
	N	1490	1491	1490	1489	1488	1490	1490
V	Correlação Pearson	-.103(**)	-.253(**)	1	-.253(**)	-.103(**)	.115(**)	.010
	Sig. (1-tailed)	.000	.000	.	.000	.000	.000	.351
	N	1489	1490	1491	1490	1489	1490	1490
V(+1)	Correlação Pearson	-.084(**)	-.103(**)	-.253(**)	1	-.253(**)	-.017	-.060(*)
	Sig. (1-tailed)	.001	.000	.000	.	.000	.255	.010
	N	1488	1489	1490	1490	1489	1489	1489
V(+2)	Correlação Pearson	.024	-.084(**)	-.103(**)	-.253(**)	1	-.026	-.019
	Sig. (1-tailed)	.177	.001	.000	.000	.	.162	.228
	N	1487	1488	1489	1489	1489	1488	1488
R	Correlação Pearson	-.023	-.081(**)	.115(**)	-.017	-.026	1	.099(**)
	Sig. (1-tailed)	.186	.001	.000	.255	.162	.	.000
	N	1489	1490	1490	1489	1488	1490	1490
R^2	Correlação Pearson	-.020	.088(**)	.010	-.060(*)	-.019	.099(**)	1
	Sig. (1-tailed)	.223	.000	.351	.010	.228	.000	.
	N	1489	1490	1490	1489	1488	1490	1490

** Correlation is significant at the 0.01 level (1-tailed).
* Correlation is significant at the 0.05 level (1-tailed).

From Table 1 we find that there is a low but significant contemporaneous positive correlation between stock return levels and trading volume. The correlation is even weaker but significant if one computes the correlations between stock returns and lagged and lead trading volume.

On the other hand, Table 1 indicates that there is no positive contemporaneous relationship between trading volume and return volatility. There is, however, a low positive but significant correlation between lagged trading volume and return volatility and a low negative correlation between lagged return volatility and trading volume. This is a first indication that there might exist a causal relationship between trading volume and return volatility. These findings are further investigated in the next sections. The results presented in this section are in accordance with previous empirical findings (BROCK; LEBARON, 1996; MESTEL; GURGUL; MAJDOSZ, 2003).

4.3. Unit Root Tests

As discussed in Section 3.3, after conducting ADF tests according to equation (2) for the time series of stock returns and trading volume we find the parameter γ to be negative and statistically significant at the sensible levels. Hence we come to the conclusion that all three time series, i.e. stock return, stock return volatility and volume (R, R^2 and V) can be considered stationary. Table 3 shows the results of the unit-root tests.

Table 3: Results of the unit root tests

Variable	ADF statistics	Critical value (1%)*
R	-29.71375	-3.4377
R^2	-14.04838	-3.4377
V	-21.33754	-3.4377

*MacKinnon critical values for rejection of hypothesis of a unit root.

4.4. Contemporaneous Relationship between Stock Returns and Trading Volume

The tests for contemporaneous relationships between stock returns and trading volume were described in Section 3.4, and they are performed by means of equations (3) and (4), which are jointly estimated by full-information maximum likelihood. The results of these are shown on Table 4.

Our findings confirm the cross-correlation analysis that there is evidence of a contemporaneous relationship between stock returns and trading volume. The parameters α_1 in equation (3) is significant at the 1% level and it is positive. There is also evidence of a lagged relationship between stock returns and trading volume, since the parameter α_2 in equation (3) is also positive, although significant only at the 6.7% level. However, the contemporaneous relationship between stock returns and trading volume is not simultaneous, since the parameter β_1 in equation (4) is not significant, which means that R depends on V, but V does not depend on R. The strong time dependency of trading volume is documented by highly significant parameters β_2 and β_3 in equation (4).

It has been often reported that price fluctuations tend to increase if there is high trading volume, especially in times of bullish markets. That is, there might be a relation between higher order moments of stock returns and trading volume. We investigated this by extending a model which relates trading volume to squared stock returns by the following regression (BRAILSFORD, 1996):

$$V_t = \alpha_0 + \phi_1 V_{t-1} + \phi_2 V_{t-2} + \alpha_1 R_t^2 + \alpha_2 D_t R_t^2 + e_t \tag{5}$$

where D_t denotes a dummy variable that equals 1 if the corresponding return R_t is negative and 0 otherwise. To avoid the problem of serially correlated residuals previously documented, we include lagged values of V up to lag 2.

After this, we find e_t in equation (5) to be largely non-serially correlated. The estimate of parameter α_1 measures the relationship between return volatility and trading volume irrespective of the direction of the price change. The estimate of α_2, however, measures the degree of asymmetry in that relationship.

Table 4: Joint estimation of equations (3) and (4)
Estimation Method: Full Information Maximum Likelihood (Marquardt)

Sample: 1 1492				
Convergence achieved after 4 iterations				
	Coefficient	Std. Error	t-Statistic	Prob.
α_0	-0.031786	0.011394	-2.789817	0.0053
α_1	0.000252	8.45E-05	2.983487	0.0029
α_2	4.74E-05	2.59E-05	1.833342	0.0669
α_3	-0.452521	0.024478	-18.48669	0.0000
β_0	155.5020	3.315029	46.90819	0.0000
β_1	144.2750	104.3029	1.383232	0.1667
β_2	-0.292207	0.015956	-18.31379	0.0000
β_3	-0.172034	0.025680	-6.699136	0.0000

Log Likelihood	-4111.776		
Determinant residual covariance	0.793950		
Equation (3): $R=\alpha_0+ \alpha_1*V+ \alpha_2*V(-1)+ \alpha_3*R(-1)$			
Observations: 1490			
R-squared	0.145265	Mean dependent var	4.82E-05
Adjusted R-squared	0.143539	S.D. dependent var	0.026101
S.E. of regression	0.024155	Sum squared resid	0.867061
Durbin-Watson stat	2.279031		
Equation (4): $V=\beta_0+\beta_1*R+\beta_2*V(-1)+ \beta_3*V(-2)$			
Observations: 1490			

Applying maximum likelihood to estimate equation (5) lead to the results shown on Table 5.

Table 5: Estimation results for equation (5)
Estimation Method: Full Information Maximum Likelihood (Marquardt)

Sample: 1 1492

Convergence achieved after 1 iterations

	Coefficient	Std. Error	t-Statistic	Prob.
α_0	155.3604	3.012434	51.57306	0.0000
ϕ_1	-0.290592	0.015486	-18.76459	0.0000
ϕ_2	-0.176159	0.022978	-7.666507	0.0000
α_1	3608.493	804.8956	4.483181	0.0000
α_2	-6288.694	1864.700	-3.372496	0.0008

Log Likelihood	-7614.723
Determinant residual covariance	1619.855

Equation (5): $V=\alpha_0+\phi_1*V(-1)+\phi_2*V(-2)+\alpha_1*R^2+\alpha_2*DUMMY*R^2$

Observations: 1489

R-squared	0.103389	Mean dependent var	106.1928
Adjusted R-squared	0.100972	S.D. dependent var	42.51888
S.E. of regression	40.31516	Sum squared resid	2411963.
Durbin-Watson stat	2.067719		

We find parameter α_1 to be positive and significant and parameter α_2 to be negative and significant. These findings strongly support the hypothesis that higher trading volume is associated with an increase in stock return volatility and that this relationship is more pronounced when stock prices increase. Good news (increasing prices) therefore induces more trading volume than bad news (declining prices), which is also consistent with the assumptions put forward by behavioral finance (RITTER, 2003).

4.5. Conditional Volatility and Trading Volume

The results of the joint estimation by maximum likelihood of equations (6) to (8) are depicted on Tables 6 and 7.

Table 6: Joint restricted estimation of equations (6) to (8)

Dependent Variable: R
Method: ML - ARCH
Date: 04/13/06 Time: 17:32
Sample(adjusted): 3 1492
Included observations: 1490 after adjusting endpoints
Convergence achieved after 13 iterations

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000179	0.000530	0.336769	0.7363
R(-1)	-0.478418	0.024741	-19.33720	0.0000
Variance Equation				
C	3.18E-05	9.75E-06	3.263173	0.0011
ARCH(1)	0.104377	0.019917	5.240555	0.0000
GARCH(1)	0.835882	0.032066	26.06789	0.0000
R-squared	0.219373	Mean dependent var		4.82E-05
Adjusted R-squared	0.217271	S.D. dependent var		0.026101
S.E. of regression	0.023092	Akaike info criterion		-4.756244
Sum squared resid	0.791884	Schwarz criterion		-4.738437
Log likelihood	3548.402	F-statistic		104.3295
Durbin-Watson stat	2.279104	Prob(F-statistic)		0.000000

Table 7: Joint unrestricted estimation of equations (6) to (8)

Dependent Variable: R
Method: ML - ARCH
Date: 04/13/06 Time: 17:33
Sample(adjusted): 3 1492
Included observations: 1490 after adjusting endpoints
Convergence achieved after 13 iterations

	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.000122	0.000476	-0.256652	0.7974
R(-1)	-0.475189	0.024236	-19.60644	0.0000
Variance Equation				
C	4.58E-06	4.28E-05	0.107059	0.9147
ARCH(1)	0.164251	0.030971	5.303439	0.0000
GARCH(1)	0.702765	0.052549	13.37358	0.0000
V	6.01E-07	3.69E-07	1.628243	0.1035
R-squared	0.219414	Mean dependent var		4.82E-05
Adjusted R-squared	0.216784	S.D. dependent var		0.026101
S.E. of regression	0.023099	Akaike info criterion		-4.751202
Sum squared resid	0.791842	Schwarz criterion		-4.729833
Log likelihood	3545.646	F-statistic		83.42729
Durbin-Watson stat	2.283296	Prob(F-statistic)		0.000000

We first estimate the parameters of equation (8) under the assumption that γ_1 is equal to 0 (restricted variance equation), which is shown on Table 6. From this we found parameters

α_1 and β_1 to be positive and significant. The observed sum ($\alpha_1+\beta_1$) is 0.93, indicating high persistence (hysteresis) in conditional volatility.

In the next step we were interested in the unrestricted conditional variance equation (Table 7). We found the estimated parameter γ_1 to be positive although significant only at the 10% level. Most interestingly, our data show a decrease in the persistence of volatility when including trading volume in equation (8), since the sum ($\alpha_1+\beta_1$) falls to 0.86 in the unrestricted regression.

As pointed out by Piscitelli et al. (1999), the term hysteresis, which comes from physics, has been used in economic and econometric theory to refer to two distinct phenomena: persistence in deviations from equilibria, possibly followed by an eventual return to a previous equilibrium state; and the presence of unit roots in systems of linear difference or differential equations, implying that a single temporary shock permanently changes the equilibrium path of the system. In empirical economics, however, hysteresis is used more loosely to mean that temporary shocks are observed to result in a persistent change from a previously persistent system state, even though this previously persistent system state cannot be verified to be in equilibrium and the persistent change cannot be verified to be permanent.

To some extent our results for the Brazilian stock market show weak support for the MDH model. Trading volume as a proxy for the flow of information has at least a weak effect on stock returns volatility. On the other hand, we found the parameters α_1 and β_1 to remain significant after including trading volume in equation (8). This can be seen as a signal that either trading volume might only be a rough proxy for the flow of information, or that the assumption of the MDH that information flows simultaneously into the market might be incorrect.

4.6. Causal Relationships

In order to test for causality we estimate a bivariate VAR(1) model, the results of which are shown on Table 8. The VAR's order was chosen based on the minimization of the Akaike and the Schwartz information criteria. From Table 8, we can see that the influence of lagged trading volume on stock returns is weak with the corresponding parameter being significant only at the 12% level (t-statistic = –1.1832). Besides, it is clear that the influence of lagged stock returns on trading volume is insignificant. However, a better picture concerning the influences of trading volume on stock returns and vice-versa can be obtained from Granger-causality tests. Table 9 reports our results of Granger testing for unidirectional causality between returns and trading volume, and between squared returns (volatility) and trading volume, respectively, based on the VAR(1) results shown on Table 8.

As expected, we cannot find evidence of a causal relationship between stock returns and trading volume in either direction. That means that short-run forecasts of current or future stock returns in most cases cannot be improved by knowledge of recent trading volume data and vice versa. In addition, Table 9 illustrates that return volatility Granger-causes trading volume and that trading volume Granger-causes return volatility even more strongly. This means that causality between trading volume and stock return volatility occurs in both directions, although more intensely from volume to volatility. This result confirms previous findings that stock price changes in any direction have information content for forthcoming trading activities (MESTEL, GURGUL; MAJSDOZ, 2003).

The bi-causal relationship between trading volume and stock return volatility can be seen as evidence that new information arrival follow a simultaneous process. This implies that the strong form of market efficiency holds since private information is reflected on stock prices.

5. CONCLUSION

In this study the empirical relationship between stock returns, return volatility and trading volume was examined by using data from the Brazilian stock market. We found evidence of a significant contemporaneous relationship between return volatility and trading volume, which is detected in the cross-correlation analysis. However, a simultaneous equation analysis show that stock returns depend on trading volume, but that this does not apply the other way round.

Table 8: VAR(1) results

Included observations: 1490 after adjusting endpoints Standard errors & t-statistics in parentheses		
	R	V
R(-1)	-0.464387 (0.02300) (-20.1874)	19.87980 (41.0352) (0.48446)
V(-1)	-1.68E-05 (1.4E-05) (-1.18372)	-0.254830 (0.02525) (-10.0916)
C	0.001830 (0.00162) (1.13095)	133.2501 (2.88583) (46.1740)
R-squared	0.220252	0.064372
Adj. R-squared	0.219203	0.063114
Sum sq. resids	0.790993	2517002.
S.E. equation	0.023064	41.14209
Log likelihood	3503.825	-7651.094
Akaike AIC	3503.829	-7651.090
Schwarz SC	3503.840	-7651.079
Mean dependent	4.82E-05	106.1866
S.D. dependent	0.026101	42.50528
Determinant Residual Covariance		0.884177
Log Likelihood		-4136.729
Akaike Information Criteria		-4136.721
Schwarz Criteria		-4136.699

Table 9: Pairwise Granger Causality Tests

Sample: 1 1492 Lags: 1			
Null Hypothesis:	Obs	F-Statistic	Probability
R does not Granger Cause V	1490	0.23470	0.62813
V does not Granger Cause R		1.40119	0.23671
R^2 does not Granger Cause V	1490	5.28956	0.02159
V does not Granger Cause R^2		12.2245	0.00049

We also find that higher trading volume is associated with an increase in return volatility and that this relationship is asymmetrical, since it is more pronounced when stock prices increase than vice versa.

Our GARCH(1,1) estimation of stock returns and volatility confirm the ARCH effects and high hysteresis in conditional volatility. The hysteresis of variance over time partly declines if one includes trading volume as a proxy for information arrivals in the equation of conditional volatility. The GARCH estimation provide an almost negligible support for the MDH (Mixed of Distribution Hypothesis), since the inclusion of trading volume in the variance equation (unrestricted estimation) produces a rather weakly significant coefficient and it does not relieve the strong ARCH effects observed in the restricted variance equation.

When it comes to Granger-causality, our results present no signs of causality between trading volume and stock returns. However, Granger causality between trading volume and return volatility is remarkably evident in both directions, although stronger from trading volume to return volatility, which indicates that information might flow simultaneously rather than sequentially into the market.

Our findings differ in some aspects from those obtained in a similar study (MESTEL, GURGUL; MAJDOSZ, 2003). While they find only weak support for a contemporaneous as well as a dynamic relationship between stock returns and trading volume, we find significant contemporaneous and dynamic relationships between these variables. One of the reasons that can explain these differences is the method used to estimate the simultaneous equations system, consisting of Equations (3) and (4), since those authors apply 2SLS, whereas we employ full-information maximum likelihood, which tends to be more robust. Another difference relates to the Granger-causality tests. While those authors find weak evidence of a causal relationship between stock returns and trading volume, we find no causality at all between these two variables. Besides, while they find that return volatility precedes trading volume, we come across some hard evidence of a mutual causality between these two variables. Of course, since the statistical and econometric methods used in both studies are similar, the differences noticed with respect to the empirical results are likely to be a consequence of different microstructure and institutional factors, which differentiate a small but developed stock market (Austria) from a relatively small but emerging stock market (Brazil).

We believe our results can help to better understand the microstructure of stock markets, especially of the emerging variety. However, since the Brazilian stock market is thin when compared to more developed markets, additional comparable investigations with respect to other markets are desirable.

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